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DISCUSSION
BRADIE ON POLANYI ON THE MENO PARADOX

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An argument of Michael Polanyi [3] for the necessity of "tacit knowledge," based upon the paradox of the Meno, is refuted correctly by Michael Bradie [1]; who observes that the paradox, in Polanyi's version, rests on the false premise that "if you know what you are looking for, there is no problem."

Bradie's refutation is based on an example, but he does not explain how the example works and why Polanyi's premise is generally fallacious. It is the purpose of this note to describe some classes of conditions under which Polanyi's premise will be false. I have given the argument less formally elsewhere [4,5], but will try to make it more precise here.

Consider a formal system, $\underline{S}$, of the usual kind (see Mendelson [2] for details): it contains a countable set of symbols, finite sequences of which are called expressions; a subset, $\underline{F}$, of the expressions are called well-formed-formulas (wffs) of $\underline{S}$, and a set of the $\underline{F}$'s are called axioms. There is a finite set, $\underline{R}$, of relations among $\underline{F}$'s called rules of inference. If there are $\underline{j}$ wffs that stand in the relation $\underline{R}_j$ to a wff $\underline{C}$, where $\underline{R}_j$ is one of the rules of inference, then $\underline{C}$ is a direct consequence of these $\underline{j}$ wffs. A proof in $\underline{S}$ is a sequence of wffs, each of whose members is either an axiom of $\underline{S}$ or a direct consequence of some of the preceding wffs by virtue of the rules of inference. A theorem, T, of S is a wff such that there is a proof in S whose final member is T.

We suppose further that there is an effective procedure to determine whether a given expression in S is a wff; that there is an effective procedure to determine whether a given wff is an axiom; and that there is an effective procedure for determining whether a given wff, C, is a direct consequence of some other given set of wffs.

A problem can now be posed for the system S by generating a wff, call it P, and setting the task of determining whether P is a theorem of S. Now P is a theorem of S iff P is the final member of some proof of S. Hence, we know exactly what we are looking for: we are looking for a sequence of wffs such that each member is a direct consequence of some previous members and P is the final member. Moreover, the effective procedures available to us enable us to determine whether any object presented to us is such a sequence.

Notice that our ability to know what we are looking for does not depend upon our having an effective procedure for finding it; we need only an effective procedure for testing candidates. Of course, even if the former procedure exists, so that the system, S, is decidable, actually finding a proof may be a non-trivial task. Hence, we can define theorem-proving problems, without tacit knowledge, in both decidable and non-decidable systems.

Since not all problems are problems of proving theorems, we wish to generalize our result. Consider a system containing wffs, as before, together with two distinct effective procedures for generating certain subsets of wffs. Call the subsets A and B. Now we can generate a member of A, and set the problem of determining whether it is also a member of B. Again, we know exactly what we are looking for: a sequence of members of B whose final member is the desired member of A. We can give to each member of A a name: the letter "A" prefixed to the number designating the order in which it will be generated by the first effective procedure. Similarly we can give to each member of B a name formed by prefixing "B" to the corresponding order number for the second effective procedure. Now we can state problems of the form: find the B-name for the wff whose A-name is An, where n is a definite number.

The particular form of naming proposed in the previous paragraph is not important to our scheme. What we require are two generators, each of which generates some set of objects that can be referred to by definite names, together with an effective procedure that, when given an object from each of the two sets, decides whether they are the same object or different objects. For example, the first generator could generate numbers named as successors of zero (e.g., 0''''''''''), while the second generator could generate numbers named by their prime factors expressed in decimal notation (e.g., 2^2x3). A simple procedure could then make the judgments of equality by recoding the former numbers decimally, performing the multiplications indicated for the latter, and comparing the recoded results for identity.

Finally, let us re-examine the example that Bradie used to refute Polanyi's premise. Bradie considers a mathematician who seeks to refute Goldbach's conjecture (that every even number is representable as the sum of two primes) by finding a counterexample: i.e., an even number that is not the sum of two primes. Any number can be named (uniquely) in decimal notation. Alternatively, it can be named (possibly non-uniquely) as a sum of two other numbers. An addition operator provides an effective procedure for determining whether a given number, named in the former fashion, is identical with a number named in the latter. Thus, the addition operator will decide that (5 + 5) is identical with 10; but that (3 + 5) is not identical with 9. Now the mathematician sets up an effective procedure for generating the even numbers, and another effective procedure for generating all pairs of prime numbers whose sum is less than some _n_. With these procedures he can now define the problem of looking for a refutation of Goldbach's conjecture: find a number, _k_, generated by the first procedure that does not belong to the numbers generated by the second procedure for _n_ = _k_. Thus we see that Bradie's example fits our general scheme for defining problem solutions prior to finding them.

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