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Data transformations in DEA cone ratio envelopment approaches for monitoring bank performances [1]

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Abstract

Cone ratio DEA (Data Envelopment Analysis) models are suggested for monitoring and/or early warning systems to be used by bank regulatory agencies. Illustrative examples are developed from data on 1984 and 1985 performances of the 16 largest banks in Texas. Five large non-Texas banks are introduced as "excellent performers" to help evaluate these Texas banks in terms of their "risk coverage" as well as "efficiency". Cone ratio envelopments are used to transform original data in order to reflect performances by the non-Texas banks with respect to "risk coverage" as well as "efficiency". Formulas for transforming to and from the original data are supplied with accompanying explanations and interpretations which include comparisons with the "risk-adjusted capital" and "risk-coverage" allowances formulas that have been adopted recently by U.S. Government (and other) regulators in banking (and insurance) in conformance with the "Basel Agreement" of 1988. © 1997 Elsevier Science B.V.

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1. Introduction

This paper reports results from research undertaken as part of an effort to explore new approaches to "early warning" and "monitoring" systems which could be used by the Texas Departments of Banking and Insurance. As used by regulatory agencies to monitor performance and to detect potential troubles, such systems generally rely on various ratios obtained from periodic accounting reports which agencies collect from the entities under their jurisdiction. These data are also sometimes synthesized into indexes of performance on the basis of various weighting or scoring schemes and supplemented, or augmented, by other reports like the CAMEL ratings which we describe below.

Brockett et al. (1994) describe results from "neural network" and other approaches to early detection of insurer insolvency. See pp. 4 ff. in Alwin (1991) for a discussion of some of the other approaches.

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[1] This is a revised version of an earlier paper by Charnes et al. (1988) which was presented at a Conference on New Uses of DEA in Management at IC[2] Institute, September 27–29, 1989, in Austin, Texas and at the University of Southern California/Deliotte and Touche Audit Symposium held at the Newporter Resort in Newport Beach, California on February 19–20, 1990. The research for this paper was partly supported by National Science Foundation Grant SES 8520806 and by the U.S. Army Contract DAKF-15-87C-0110 with the Center for Cybernetic Studies at The University of Texas. It was also partly supported by the IC[2] Institute of The University of Texas at Austin.

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Here we focus on banks and develop a simplified (illustrative) example based on extensions of Data Envelopment Analysis (DEA) which have been accorded the name "Cone Ratio (CR) Envelopments".

This extension of DEA makes it possible to adjust the originally collected data in ways that can take account of a variety of complex considerations which are otherwise difficult to treat. We provide an illustration of one possible approach which uses the dual evaluators from the multiplier problem of DEA to adjust data in ways that reflect the performances of excellent banks. The thus adjusted data can then be used to effect evaluations of other banks in dimensions such as "efficiency" and "risk coverage" to see whether (and by how much) they fall short of excellent performance.

We have used the term "excellent" rather than the term "efficiency" which is customary in DEA because we want our evaluations to include "risk coverage" as well as "efficiency". This allows for possibilities in which accounts like "provisions for loan loss reserves" may require augmentation, as we will show in our example, even when this will *worsen* the efficiency of the bank being evaluated.

Our study was conducted in the mid and late 1980's. It is therefore of interest to note that U.S. Government bank regulatory agencies subsequently (in the early 1990s) adopted an approach that also involves data adjustments to obtain a value which is referred to as "risk-adjusted capital" *en route* to determining whether the "risk coverage" is adequate. The National Association of Insurance Commissioners has also adopted a similar approach but we now focus on banks where these approaches to "risk-adjusted capital" flowed from an agreement concluded in Basel, Switzerland, in 1988, which was subsequently adopted by U.S. Government (and other) regulatory agencies to improve the treatments of "risk" in their monitoring systems. All data used in these adjustments are deterministic, however, so we will refer to this as "risk coverage". The analogy we are using is with an insurance policy in which one *knows* the types and amounts of coverage that will be provided if one or more of the thus insured events occurs. For discussion and definitions of "risk" and "risk coverage" see Brockett et al. (1992) and Brockett et al. (1996).

For considerations like those introduced by the Basel agreement, there is another element to be considered in that these involve establishing lower bounds for use in evaluating the risk coverage that is provided. This brings us to another development in DEA in the form of what are referred to as "Assurance Region (AR)" approaches, which can be used to establish bounds on admissible *solutions* as an alternative (or supplement) to the *data transformations* used in our CR approach. These AR approaches proceed indirectly in a manner that is related to the "multiplier" rather than the "envelopment problem". However, this can be given an interpretation in terms of cones which relate the AR to the CR envelopment approaches and this opens possibilities for use that differ from the ones like the approaches to risk-coverage evaluations that are currently used in bank regulatory activities.

We do not try to cover all aspects of these topics in the present paper. In the next section, we provide some background on the context in which this work was performed. Then we describe aspects of the rating systems used to help identify problem banks and to help guide bank examinations when in-the-field activities seem warranted. The inputs and outputs we use are then described. This is followed by a formal development of the "CR" approaches after which the AR approaches are described. The Texas banks and the excellent (nonTexas) banks used in this study are next described along with reasons for choosing these banks. Example applications are then discussed along with the controls used and the results secured after which a final section returns to a discussion of the Basel accords and compares its very rigid approach with the more flexible one we are suggesting.

2. Background

To help Banking Department (and other) personnel develop a "feel" for what DEA might be able to do, we confined ourselves to very few inputs and outputs and used only very few (well known) banks. We also selected the period 1984–1985 for our illustrative example for reasons like the following: R.D. Rieke (1989) reported that the years prior to 1984 were very good ones for banking in Texas but the situation began to deteriorate in that year by reference to both an increasing number and an in-

Table 1
Number of bank failures (through 10-31-88)

Year	Texas		U.S.	
	State banks	National banks	Total Texas	Nationwide
1980	0	0	0	10
1981	0	0	0	10
1982	4	3	7	42
1983	1	2	3	48
1984	2	4	6	79
1985	5	7	12	120
1986	14	12	26	145
1987	24 *	28	52	188
1988	39	62	101	175 **

* Includes 2 private uninsured banks.
** Includes 40 banks (including 9 State-chartered banks) which were closed, with some reopening under a different aegis as a result of FDIC assistance.
Source: *Audit of the Examination and Enforcement Functions of Texas Department of Banking*, A report to the Legislative Audit Committee (Austin, Texas, Office of the State Auditor, January 1989). See Alwin (1989).

Table 2
Number of banks closed because of financial difficulties 1943–1987

Year	Total	Year	Total	Year	Total
1943	5	1958	9	1972	3
1944	2	1959	3	1973	6
1945	1			1974	4
1946	2	1960	2	1975	14
1947	6	1961	9	1976	17
1948	3	1962	3	1977	6
1949	9	1963	2	1978	7
		1964	8	1979	10
		1965	9		
1950	5	1966	8	1980	10
1951	5	1967	4	1981	10
1952	4	1968	3	1982	42
1953	5	1969	9	1983	48
1954	4			1984	79
1955	5			1985	120 *
1956	3	1970	8	1986	145 **
1957	3	1971	6	1987	203 ***

Source: *1986 Annual Report*, Federal Deposit Insurance Corporation (FDIC), Washington, DC.
* Includes one bank granted financial assistance.
** Includes seven banks granted financial assistance to prevent bank failure under Section 13(c)(1) of the Federal Deposit Insurance Act.
*** Includes 19 banks granted financial assistance. (This information was obtained by telephone on January, 1988, from FDIC Office of Information in Washington, DC.)

creasing proportion of bank failures in Texas relative to the rest of the country. This suggested that the transition from 1984 to 1985 could provide a good start in testing DEA (or any other system) for use in monitoring bank performance in Texas.

The data on bank failures exhibited in Table 1 show that the relative proportion of Texas bank failures climbed to nearly 58% of the total number of U.S. bank failures in the United States in 1988. Tables 2 and 3 supply further background. Allowing for asset size, as in Table 3, the apparent tendency of a decrease in the average value of assets involved is portrayed in a different light by reference to the median values given in the last row which show (a) a relatively steady median value, but with an aberration in 1983, and (b) a relation between means and medians which suggests a statistical distribution that is skewed to the right. We therefore infer that banks larger than average must generally be involved in these failures and this suggested that illustrations might best be centered on relatively large banks.

3. CAMEL Ratings

We describe certain aspects of the evaluation systems employed in the U.S. and their use in identi-

Table 3
Failed bank statistics 1982–1987

	1982	1983	1984	1985	1986	1987
Number of bank failures	42	48	79	120	145	203
Commercial banks	34	45	78	118	144	203
Savings banks	8	3	1	2	1	0
Failure rate	0.28%	0.33%	0.54%	0.81%	0.98%	1.45%
Average asset size (millions)	$277.0	$146.4	$41.5	$73.5	$53.0	$46.7
Median asset size (millions)	$18.5	$24.9	$19.6	$17.3	$21.1	N/A

Source: Adjusted data from FDIC Call reports.

fying problems and guiding activities such as the field examinations that the Banking Department in Texas must conduct. The U.S. Government's Federal Deposit Insurance Corporation (FDIC) developed a bank rating system in the early 1970's which is referred to as the CAMEL Ratings (*C*apital, *A*sset, *M*anagement, *E*quity, *L*iquidity). Also called Uniform Interagency Bank Rating System by the Texas Department of Banking — see Appendix C in Alwin (1989) — each component is rated on a scale of 1 to 5 as a basis for identifying problem banks. Generally speaking, these ratings are obtained from bank examiners on the basis of information obtained in their audits. Taking all ratings into account, an examiner is then supposed to apply his best judgment to obtain an "overall" rating. This can have drawbacks. For instance, an examiner may derive component values *after* providing his or her overall rating (instead of vice versa) and the component ratings, in turn, need not be independent of each other. In any case, these ratings are then incorporated with other information obtained from the Call Reports of the FDIC as well as the U.S. Office of the Comptroller of the Currency, Board of Governors or the Federal Reserve System and Federal Financial Institutions Examination Council. These "Call Reports" are collected and regularly processed as part of the Texas State Banking Department's system for identifying problem banks.

The bar chart portrayed in Fig. 1 supplies infor-

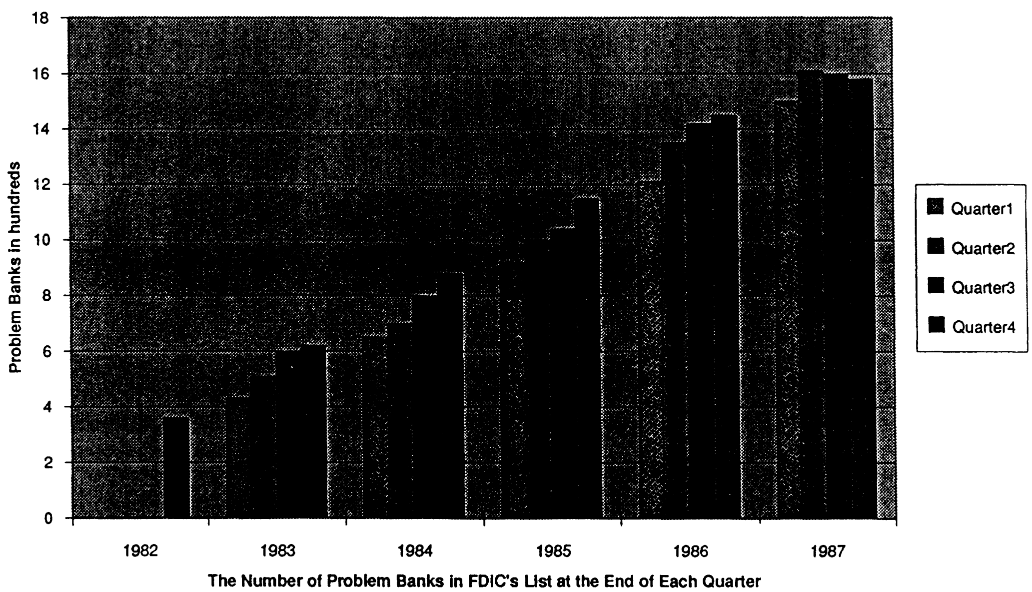

Fig. 1. Growth of FDIC problem list.

mation on the number of problem banks identified by quarter for each year from 1983 to 1987. Evidently, the number of problem banks grew substantially during this period and this resulted in a large increase in the workloads of bank examiners.

The CAMEL rating system can supply additional insight into effects on the productivity of bank examiners occasioned by increases in the number of problem banks. (See Barr et al. (1994a) for their use as a component in early warning systems.) Fig. 2 summarizes the situation for Texas in 1986 and 1987. Evidently CAMEL 1 ratings (the most favorable value) were associated with approximately a 27% dropoff in assets examined per examiner hour from 1986 to 1987. This is perhaps tolerable, but the Texas State Auditor's Office in its review of the Banking Department's performance highlighted even more serious dropoffs of 63% and 38%, respectively, in the CAMEL 4 and 5 ratings (the least favorable ratings) when going from 1986 to 1987.

Still more serious than this dropoff in assets examined per hour, as noted by the Texas State Auditor, is the fact that 13 of the 24 CAMEL 1 rated banks and 32 of the 55 CAMEL 2 rated banks had not been examined in the past two years at the time of their audit (1988–1989). Furthermore, the overload on audit staff also resulted in other failures to examine. In fact, during 1988, 25 out of 158 CAMEL 3 rated banks, 22 out of 143 Camel 4 rated banks and 7 out of 45 CAMEL 5 rated banks were not examined even though the Texas Department of Banking is required by law to examine all CAMEL 3, 4, and 5 rated banks annually and all CAMEL 1 and 2 rated banks at least once every 24 months. See Alwin (1989).

4. Data envelopment analysis and bank selection

We now turn to DEA and the uses that might be made of it. Introductory treatments of DEA and its uses have been supplied elsewhere. See e.g., Banker et al. (1989), or Charnes et al. (1993). We therefore concentrate on extensions of the original formulations which appear to be particularly attractive for use of DEA in the kinds of monitoring and information systems we are considering. In particular, we may note that there is a need in such systems for somehow combining data that might be reported systematically and objectively (e.g., data obtained from the periodic FDIC Call Reports) into a monitoring system that will (a) supply some kind of overall evaluation score for each candidate bank in terms that might relate them to other possible candidates, and (b) identify possible sources of trouble not only for use (i) as justification for an onsite audit but also

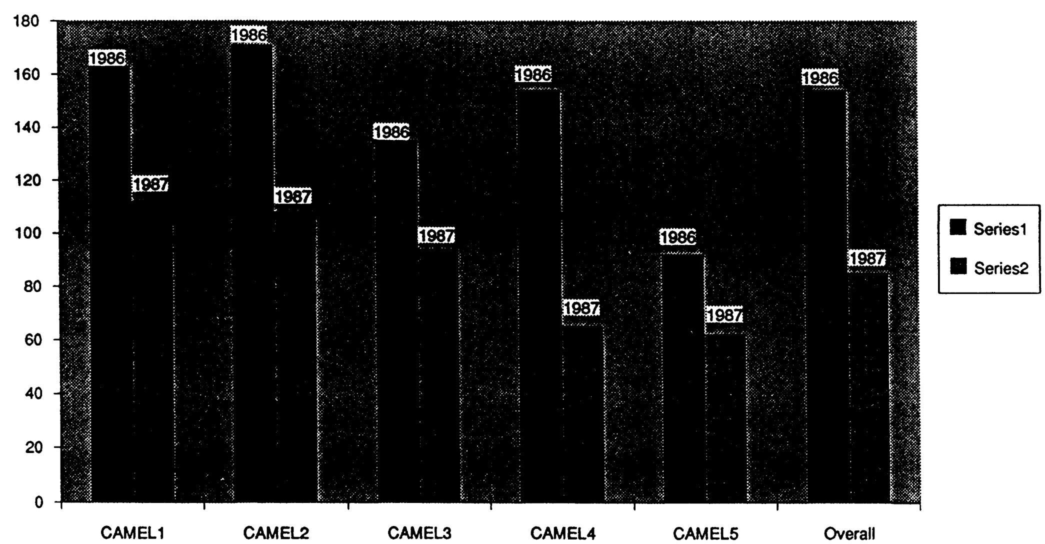

Fig. 2. Examination division efficiency assests examined per examiner hour ($000) by CAMEL rating.

for use (ii) in identifying potential trouble spots to help guide such audits in each of the banks that might be examined.

The first DEA application to banking was undertaken by Sherman and Gold (1985) who used it to evaluate the performance of branches of a parent bank from an internal management standpoint. Another application, as reported in Charnes et al. (1990), was more closely related to the problem of regulatory monitoring and audit guidance for use by an external regulatory agency with major attention being devoted to a new "cone ratio DEA" approach which, *inter alia*, (a) dealt with the problem of "too many efficient DMUs" which were noted by Sherman and Gold (1985) and (b) made it possible to exploit a priori information that could be made available for these monitoring purposes.

Here we will adapt the cone ratio approach in the following manner. First, we select 16 banks headquartered in Texas and five from other states to obtain a total of 21 banks. For the reasons noted in our discussion of Table 3, all of the banks which we selected are large in size — being listed among the 300 largest U.S. commercial banks being listed by *Business Week* as having total assets in excess of $1 billion each during 1983–1985. Only 16 Texas banks belonged in this class and this determined our choice of Texas banks. The five non-Texas banks in the total of 21 banks used in this study were selected because they are widely regarded as being excellent banks and so we use them to explore the cone ratio approach for its ability to bring evidence from these excellent banks to bear for evaluating the performances of the Texas banks included in our study.

5. Inputs and outputs

The survey article by Allen Berger and David Humphrey which opens this special issue of the *European Journal of Operational Research* provides a detailed discussion of issues involved in choosing the inputs and outputs to be used for evaluating bank performances. Here, however, our intention is to illustrate the approach to data adjustment we are proposing and the kinds of data mixes it can handle. Hence we confine ourselves to the four inputs and four outputs that are listed in Table 4. These differ from those used in Sun (1988) and in Charnes et al. (1990) which were confined to flow (income-statement type) items only. Here a mix of flow and stock (balance-sheet type) items are used because, as already set forth, we want to evaluate risk coverage as well as efficiency. (See also Barr et al. (1994).)

Table 4
Inputs and outputs

INPUTS
Interest Expense
Interest expenses on deposits
Expense for federal funds purchased and repurchased in domestic offices
Non-Interest Expense
Salaries and employees benefits
Occupancy expense, furniture, and equipment
Provision for Loan Losses
Total Deposits
Sum of interest bearing and noninterest bearing deposits
OUTPUTS
Interest Income
Interest and fees on loans
Income on federal funds sold and repurchases in domestic offices
Total Non-Interest Income
Allowances for Loan Losses
Total Loans
Loans, net of unearned income

Source: Adjusted data from FDIC call reports are used in this study.

Reasons for choosing the items used in this study can be summarized as follows. Interest Expense and Non-Interest Expense appear to be reasonable as inputs for use in evaluating managerial efficiency and, in any event, these choices have been justified (or at least explained) in Sun (1988) and in Charnes et al. (1990). Provision for Loan Losses is included as an input on the usual justification in "accrual accounting" that this should be regarded as an expense (i.e., an input) for the period in which the loans were made rather than in the period when the loss occurs. See Welsch and Zlatkovich (1989) as well as the formal definitions of "accrual" and "expense" in Cooper and Ijiri (1983). This accounting entry directly affects the balance sheet (as well as the income statement) in a way that is pertinent to "risk coverage". Hence we include it even though, as Keeton (1989) notes, the procedures flowing from the Basel Agreement are directed to balance sheet items only insofar as "risk coverage" is concerned. Finally, Total Deposits (a liability item on the balance sheet) is included as an input on the usual supposition (from economics) that banks are in the business of transforming deposits into loans. That is, banks (as distinct from other businesses) serve the "intermediation" function of transforming deposits into loans. See Colwell and Davis (1992). Here, however, we also want to use this account for its bearing on the "risk coverage" that is needed. In any case, we have a mixture of stocks and flows to be considered as inputs and this mixture is enough to serve our purposes so we refrain from adding the many more accounts with accompanying increases in complexity that might be pertinent in an actual application.

Turning from inputs to outputs, we can again observe that both Interest Income and Total Non-Interest Income as "outputs" are flow items of the kind which are commonly used to evaluate bank performance in terms of their contribution to current earnings. The use of Loans (a balance sheet item) as an output is justified along lines similar to the ones used at the conclusion of the preceding paragraph where we explained why we were treating deposits as an input.

As usual, Allowance for Loan Losses — a contra asset valuation account — consists of the beginning balance plus the current provision for loan losses (as explained in the discussion of inputs) minus any write-offs for "bad loans" during the period plus restoration of amounts previously charged to this reserve as uncollectible. The inclusion of this allowance as an output is undertaken here because measures of risk do not appear to be directly available from the Call Reports. In particular, we regard this allowance as a provision for "risk insurance" or, more precisely, as an allowance for covering potential risks. The usual accounting treatment of Loan Loss Allowances as a "valuation reserve" to be deducted from loans receivable, might seem to indicate that this insurance is provided for loans. (See the definitions of "valuation accounts" in Cooper and Ijiri (1983).) It is better viewed, however, as a form of insurance to cover risks that depositors might be exposed to if these allowances were not made (via the income statements) in each period and, in fact, this is reflected in the accounting treatment which is directed to preventing an increment to Net Worth that might otherwise be available for distribution to stockholders. This, in any event, is the way we are interpreting the Allowance for Loan Losses with DEA then being used to compare the adequacy of each bank's reserves relative to others with similar loans, deposits, expenses, etc., which are also found to be "DEA efficient" or more generally, are found to be "excellent" after the data have been suitably adjusted. Finally, we use the same reasons as before — viz., our use of this as an illustrative example — and restrict our outputs to these very few items, just as we did for the few inputs we used.

6. Cone ratio DEA models

We are examining how DEA might be used as part of a data system for monitoring the performance of individual banks — both with respect to efficiency and risk coverage — by using the data (and the banks) we have already described. To deal with these issues we will have recourse CR extensions to the CCR model in DEA. The cone ratio approach, as will be seen, makes it possible to increase both the power and the flexibility of DEA by recourse to ancillary information to effect data adjustments which can be brought to bear in effecting evaluations from the allowable solutions.

We start with the following formulation of a CCR model which we recast into a cone ratio formulation in the following multiplier form — i.e., as described in Banker et al. (1989) we use the linear programming model that is dual to the usual envelopment (primal) form

$$
\begin{aligned}
&V_p = \text{Max } \mu^{\mathrm{T}} Y_0 \\
&\text{s.t.} \\
&-\omega^{\mathrm{T}} X + \mu^{\mathrm{T}} Y \le 0, \qquad (1)\\
&\omega^{\mathrm{T}} X_0 = 1, \\
&\omega \in V,\ \mu \in U.
\end{aligned}
$$

A detailed technical development of these cone ratio models may be found in Charnes et al. (1990)

— see also Charnes et al. (1989) — so we here only sketch some of its main features as follows. First, we note that the nonnegativity and positivity conditions which usually apply to the solutions allowed in CCR ratio models are here replaced by the more general conditions noted at the bottom of (1) for the solution vectors ω and μ. In particular, we require $V \subset E_{+}^{m}$, $U \subset E_{+}^{s}$, which are closed convex cones, to be in the m-dimensional positive orthant E_{+}^{m} for the ω vectors associated with the inputs and in the s-dimensional positive orthant E_{+}^{s} for the μ vectors associated with the outputs. If $V \subset E_{+}^{m}$ and $U \subset E_{+}^{s}$ with $V \cap$ Boundary of $E_{+}^{m} = \varnothing$ and $U \cap$ Boundary of $E_{+}^{s} = \varnothing$, then all nonefficient DMUs on the frontier can be identified without the use of the non-Archimedean elements that enter into standard versions of the CCR model. However, if we set $V = E_{+}^{m}$ and $U = E_{+}^{m}$ then the usual CCR conditions apply including the non-Archimedean elements used to guarantee positivity of the components of ω and μ.

We are using X and Y in (1) to represent the $m \times n$ and $s \times n$ matrices of observed inputs and outputs, respectively, for the n decision making units (DMUs) which are to be considered with X_0 and Y_0 representing the input and output vectors of observed values for the DMU to be evaluated. We also use X_j and Y_j, respectively, to represent the vectors of inputs and outputs for the jth DMU and we assume that $X_j \in \text{Int}\,(-V^*)$, $Y_j \in \text{Int}\,(-U^*)$ for any j. That is, we assume that these vectors are in the interiors of the "polar cones" $-V^*$, $-U^*$ associated with V and U, where $\text{Int}(-V^*) = \{v\colon v^{T}\bar{v} \geq 0$ for all $\bar{v} \in V$ and $\bar{v} \neq 0)$ and $\text{Int}\,(-U^*) = \{u\colon u^{T}\bar{u} > 0$ for all $\bar{u} \in U$ and $\bar{u} \neq 0\}$. See Chapters VII and VIII in Charnes and Cooper (1961) for a compact development of the theory of convex cones in relation to linear programming. See also D. Gale (1951).

With these definitions in hand, we can write the dual to (1) in the following form:

$$V_D = \text{Min}\ \theta$$

s.t.

$$-X\lambda + \theta X_0 \in -V^*,$$

$$Y\lambda - Y_0 \in -U^*,$$

$$\lambda \geq 0, \tag{2}$$

where, as usual, θ is not restricted in sign but the components of λ are all constrained to be nonnegative as required by the condition $\lambda \geq 0$ imposed on this vector of variables. The notation in (2) indicates that the solutions generated by these λ and θ choices are required to be in the polar cones, $-V^*$ and $-U^*$. I.e., we must have $-X\lambda + \theta X_0 \in -V^*$ and $Y\lambda - Y_0 \in -U^*$. If $V = E_{+}^{m}$ and $U = E_{+}^{s}$ then the ordinary expression for the dual problem in the CCR ratio form will apply — viz., $X\lambda + \theta X_0 \geq 0$ and $Y\lambda - Y_0 \geq 0$.

To complete the part of this development that is immediately needed for our use of the cone ratio approach, we note that we can define our cones in different ways — as context and convenience may suggest. For instance, we may form V and U from suitably chosen matrices A and B via

$$V = \{A^{T}\alpha\colon \alpha \geq 0\}, \tag{3}$$

where $A^{T} = (a^1, a^2, \ldots, a_l)$, $\alpha \in E_{+}^{l}$, $a^i \in E_{+}^{m}$, $i = 1, \ldots, l$; and

$$U = \{B^{T}\gamma\colon \gamma \geq 0\}, \tag{4}$$

where $B^{T} = (b^1, b^2, \ldots, b^k)$, $\gamma \in E_{+}^{k}$, $b^r \in E_{+}^{s}$, $r = 1, \ldots, k$, by using the property that the corresponding cones consist of the sums of all nonnegative multiples of the vector $(a^1, a^2, \ldots, a^l)$ for A and $(b^1, b^2, \ldots, b^k)$ for B which are in the positive orthants defined by E_{+}^{m} and E_{+}^{s}, respectively.

How to choose the matrices A and B for use in our bank monitoring system will be discussed in the next section. Here we only note that these same A and B matrices may be used to generate the polar cones represented in (1) and (2) via

$$V^* = \{v\colon Av \leq 0\}, \tag{5}$$

$$U^* = \{u\colon Bu \leq 0\}, \tag{6}$$

to obtain

$$-V^* = \{v\colon Av \geq 0\}, \tag{7}$$

$$-U^* = \{u\colon Bu \geq 0\}, \tag{8}$$

where $v \in E^{m}$ and $u \in E^{s}$.

By means of Eqs. (3) and (4) we can transform (1) to the following new problem:

$$\begin{aligned} &V_{p1} = \text{Max } \gamma^{\mathrm{T}}(BY_0) \\ &\text{s.t.} \\ &-\alpha^{\mathrm{T}}(AX) + \gamma^{\mathrm{T}}(BY) \le 0, \qquad (9) \\ &\alpha^{\mathrm{T}}(AX_0) = 1, \\ &\alpha^{\mathrm{T}} \ge 0,\ \gamma \ge 0;\ \alpha \in E^l \text{ and } \gamma \in E^k. \end{aligned}$$

In fact, setting $\bar{X} = AX$ and $\bar{Y} = BY$, we see that this transforms the original data and yields a new problem which is in the same form as (1) — viz.,

$$\begin{aligned} &V_{p1} = \text{Max } \gamma^{\mathrm{T}}\bar{Y}_c \\ &\text{s.t.} \\ &-\alpha^{\mathrm{T}}\bar{X} + \gamma^{\mathrm{T}}\bar{Y} \le 0, \qquad (10) \\ &\alpha^{\mathrm{T}}\bar{X}_0 = 1, \\ &\alpha \ge 0,\ \gamma \ge 0, \end{aligned}$$

where the solution vectors α and γ which are constrained to be nonnegative now replace the vectors ω and μ in (1) which are constrained to lie in the closed convex cones V and U as defined for (1), above, and where $\bar{X}$, $\bar{Y}$ and $\bar{X}_0$, $\bar{Y}_0$ replace the data originally represented in (1).

By means of Eqs. (7) and (8) we can, similarly, transform (2) to

$$\begin{aligned} &V_{D1} = \text{Min } \theta \\ &\text{s.t} \\ &-(AX)\lambda + \theta(AX_0) \ge 0, \qquad (11) \\ &(BY)\lambda - (BY_0) \ge 0, \\ &\lambda \ge 0, \end{aligned}$$

which we can also represent as

$$\begin{aligned} &V_{D1} = \text{Min } \theta \\ &\text{s.t.} \\ &-\bar{X}\lambda + \theta\bar{X}_0 \ge 0, \qquad (12) \\ &\bar{Y}\lambda - \bar{Y}_0 \ge 0, \\ &\lambda \ge 0. \end{aligned}$$

Via (10) and (12), we see that these problems are in the usual primal and dual linear programming form for a CCR Data Envelopment Analysis. How to get back to solution values in terms of the original data represented in the X and Y matrices will be discussed below.

7. Assurance region

As noted in the preceding section, the cones U and V may be defined and used in dual ways. A different cone representation possibility could proceed in the manner suggested by the "AR" approach developed by Thompson and Thrall and their collaborators (Thompson et al., 1986, 1990, 1994). We turn aside for a moment to examine these possibilities for which we introduce the following conditions as taken from Thompson et al. (1986) in which this approach was originally published,

$$\underline{k} \le \frac{w_i}{w_r} \le \bar{k}, \quad \text{or} \qquad (13)$$

$$\underline{k}w_r \le w_i \le \bar{k}w_r, \qquad (14)$$

Here we are restricting the variables in (1) to satisfy these additional conditions where $\underline{k}$ and $\bar{k}$ are prearranged lower and upper bounds.

We can also accord the AR approach a geometric interpretation in terms of cones. To see that the condition (13) or (14) defines a cone we need only note that (a) these w_i and w_r pairs are coordinates of a solution point and that (b) multiplication of all coordinates by any positive constant, c, leaves these inequalities unaltered in (14). As can be seen from (13) and (14), the AR approach places explicit constraints on the admissible values of the dual variables but does not transform the data. Our cone ratio envelopment approach proceeds in the opposite direction. It transforms the original data but does not otherwise introduce new constraints in the dual or new variables in the primal.

The AR approach also extends the original DEA formulations in ways that can provide added flexibility along with more precise controls on the solutions. See also the discussion in Cooper et al. (1996) which shows how the AR approaches can be used to relax the very tight conditions that are needed for valid use of the allocative efficiency models in DEA. It has disadvantages, however, in that it may require a great deal of knowledge (or information) in order to impose the requisite conditions on all of the pertinent w_i, w_r pairs that might need to be considered in *each* of the $j = 1, \ldots, n$ DMUs to be evaluated, and in other cases very complex interactions may need to be allowed for. The resulting constraint sets may also be large and unwieldy and introduce requirements for computation that require extensions to available DEA codes. (However, see Ali (1993). See also Charnes et al. (1990) for simple treatments of conversions between an assurance region approach and a coneratio approach.)

How to proceed from this cone form to the forms described in the preceding section are discussed in detail in Appendix A. See also Charnes et al. (1991). As is apparent from (10) and (12), however, nothing more is required for the thus transformed data than to use already available DEA computer codes in order to obtain the solutions which are of interest and their associated computer printouts. Finally, the CR approach can also be used to reduce possible strains on available knowledge that can accompany a reliance on only AR approaches with inequality bounds to be imposed on each of (possibly) many variables involved in particular applications.

8. Empirical implementation and use

We illustrate one way in which these models may be used by introducing 5 model banks (which in this case are non-Texas banks) into the collection of banks used to evaluate the performance of our 16 Texas banks. These five non-Texas banks are Bankers Trust, Citibank, Morgan Guaranty, Wachovia and First Interstate of Nevada. Selected in collaboration with staff of the State Banking Department and checked with various banking experts, this collection of "excellent" banks is subjected to further tests such as the following. First the entire collection of 21 banks is to be submitted to an ordinary DEA analysis which here takes the "CCR ratio form", as described in Charnes et al. (1993). The results are then reviewed to see whether the proposed "excellent" banks prove to be efficient relative to the banks to be evaluated as well as relative to each other. Customary tests of sensitivity and the use of window analyses, etc., can also be applied to evaluate these results. The surviving subset of efficient banks is then used to provide the required cone of constraints in a manner that we now develop.

It should be noted that the steps we have just described will provide us with optimal dual variable values for these five banks. These optimal dual variables can then be used to obtain a new matrix

$$D = \begin{bmatrix} B & 0 \\ 0 & A \end{bmatrix} \tag{15}$$

which can be applied in the manner suggested for the matrices B and A in going from (11) to (12) in the preceding section of this paper. There is a problem to be considered in that the optimal dual variables need not unique in which case one might study the consequences of using one or more such alternate optima en route to arriving at a preferred one. We do not deal with this problem here, however, since (a) this would complicate our development and (b) it is not pertinent for our example. This latter proposition follows from the fact that for a basic optimal solution, if all relevant coefficients of the linear objective function associated with nonbasic variables are nonzero, then the optimal solution is unique (Thrall, 1996). This is the case for our numerical example.

These dual variable values are used to generate cones within which the dual variable values of the banks to be evaluated must lie. These cones reflect the "risk coverages" as well as the "efficiencies" exhibited by these "excellent" banks in the input and output accounts we have selected for this purpose. Because the conditions of excellence are assigned to them on an a priori basis, we also need to be able to ascertain whether the initially designated banks fulfill these conditions. We therefore adopt a two-stage procedure and use the first stage to ascertain whether these conditions are satisfied.

Table 5 can help us to put the strategy described in the immediately preceding paragraph in more concrete terms. As can be seen, although Morgan Guaranty is supposed to be "excellent", it fails the DEA efficiency test in 1984 and hence is eliminated from candidacy as a member of the spanning cones to be used for our D matrix in that year. In 1985, however, all 5 members of the excellent bank set survive this part of the efficiency CCR test and are therefore candidates that can be used to generate the spanning cones that are to be used in evaluating the performances of our Texas banks.

We can now apply our cone ratio envelopments flexibly as follows. In 1984 we remove Morgan Guaranty from its status as "excellent" but retain it for use in 1985. We also experiment further by reducing the number of excellent banks from 5 to 3 by removing Citibank as well as Morgan Guaranty from excellent status in 1984 as well as Wachovia and Citibank in 1985.

Table 5
Results from the CCR model

No.	Bank title	Efficiency score	Reference banks					Sum of slacks (in $1 000 000)
1984								
1	BANKERS TRUST CO	1.0000	1	11	12	14		0
2	CITIBANK N A	1.0000	1	2	4	6	21	0
3	MORG. GUARANTY TR CO NY	0.9757	1	6	12	14	15	385.04
4	WACHOVIA BK and TR CO NA	1.0000	1	4	9	21		0
5	INTERFIRST BK AUSTIN NA	1.0000	5	6	12	14		0
6	TX COMM BK AUSTIN NA	1.0000	6					0
7	FIRST CITY BK OF DALLAS	1.0000	5	7				0
8	INTERFIRST BK DALLAS NA	1.0000	8	14	16			0
9	MBANK DALLAS NA	1.0000	6	9	12	15	18	0
10	REPUBLIC BK DALLAS NA	0.9687	6	8	12	14	15	758.03
11	INTERFIRST BK FT WOR NA	1.0000	1	11	12			0
12	TX AMER. BK FT WOR. NA	1.0000	6	7	12			0
13	ALLIED BK OF TX	0.8373	6	8	11	12	15	591.83
14	CAPITAL BK NA	1.0000	7	14				0
15	FIR CITY NAT BK HOUSTON	1.0000	6	8	11	15	16	0
16	INTERFIR BK HOUSTON NA	1.0000	6	7	14	16		0
17	REPUBLIC BK HOUSTON NA	0.7794	6	15				461.56
18	TX COMMERCE BK NA	1.0000	5	6	14	18		0
19	FROST NAT BK SAN ANTONIO	1.0000	19	21				0
20	NB OF COMM SAN ANTONIO	0.9044	6	14	21			145.14
21	FIRST INTRST BK NEVADA NA	1.0000	7	21				0
1985								
1	BANKERS TRUST CO	1.0000	1	3	8	21		0
2	CITIBANK N A	1.0000	2	6	15	21		0
3	MORG. GUARANTY TR CO NY	1.0000	1	3	8	21		0
4	WACHOVIA BK and TR CO NA	1.0000	4	8	15	21		0
5	INTERFIRST BK AUSTIN NA	1.0000	5	6	8			0
6	TX COMM BK AUSTIN NA	1.0000	6	7	21			0
7	FIRST CITY BK OF DALLAS	1.0000	7					0
8	INTERFIRST BK DALLAS NA	1.0000	8	15	16	21		0
9	MBANK DALLAS NA	1.0000	6	8	9	12		0
10	REPUBLIC BK DALLAS NA	1.0000	5	8	10			0
11	INTERFIRST BK FT WOR NA	0.9464	3	4	6	7	21	91.4
12	TX AMER. BK FT WOR. NA	1.0000	6	8	9	12		0
13	ALLIED BK OF TX	0.8158	6	8	21			791.84
14	CAPITAL BK NA	1.0000	8	14				0
15	FIR CITY NAT BK HOUSTON	1.0000	15	21				0
16	INTERFIR BK HOUSTON NA	1.0000	7	16	21			0
17	REPUBLIC BK HOUSTON NA	0.8585	4	5	6	10		336.94
18	TX COMMERCE BK NA	1.0000	8	16	18			0
19	FROST NAT BK SAN ANTONIO	0.9568	1	4	21			390.84
20	NB OF COMM SAN ANTONIO	0.9621	5	6	21			100.76
21	FIRST INTRST BK NEVADA NA	1.0000	7	21				0

For this example the matrix D in Eq. (15) is therefore formed from 3 banks, each with 4 outputs and 4 inputs with dual variables values arranged in the following 6×8 matrix,

$$D = \begin{bmatrix} \mu_{11}^* & \mu_{21}^* & \mu_{31}^* & \mu_{41}^* & 0 & 0 & 0 & 0 \\ \mu_{12}^* & \mu_{22}^* & \mu_{32}^* & \mu_{42}^* & 0 & 0 & 0 & 0 \\ \mu_{13}^* & \mu_{23}^* & \mu_{33}^* & \mu_{43}^* & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \omega_{11}^* & \omega_{21}^* & \omega_{31}^* & \omega_{41}^* \\ 0 & 0 & 0 & 0 & \omega_{12}^* & \omega_{22}^* & \omega_{32}^* & \omega_{42}^* \\ 0 & 0 & 0 & 0 & \omega_{13}^* & \omega_{23}^* & \omega_{33}^* & \omega_{43}^* \end{bmatrix} \tag{16}$$

The stars in this matrix indicate that these dual variable values are optimal, with the μ^* in the first three rows being the values applicable to the four outputs and the ω^* values in the last three rows being applicable to the four inputs. These optimal dual variable values (which are unique) are associated with the outputs (first 3 rows) and inputs (last 3 rows), respectively, for Bankers Trust, Wachovia and First Interstate Nevada in 1984 and Bankers Trust, Morgan Guaranty and First Interstate Nevada in 1985. The resulting variable values are to be used to transform the data and to generate the cones within which evaluations are to be conducted where, we note, an interchange in the first 3 rows and the corresponding interchange in the last 3 rows will affect neither the cone structures of U and V nor the dual problems in (9) and (11). It will change only the order of the constraints in (11) and the order of the components in the new input and output vectors $\overline{X}_j$ and $\overline{Y}_j$.

9. Cone ratio transformations and results

As shown in Table 6, the numerical values of these optimal dual variable values in the D matrix are allowed to change from year to year in order to reflect corresponding changes in bank performances. These different matrices are then applied to the original data in each year in order to effect a DEA analysis with the thus transformed data to achieve results like those that are summarized in Table 7. Comparison with Table 5 shows a drastic reduction in both years in the number of banks that achieve a rating of unity with all slacks zero, as required for fully efficient performance. The excellent banks used to form our cones continue to maintain their status. That is, we do not now experience an elimination from excellence status such as occurred for Morgan Guaranty with the original (untransformed) 1984 data in Table 5. Although some tradeoff occurs between the efficiency scores and slack amounts for Morgan

Table 6
Spanning matrices for 1984 and 1985 cones

1984 (Banks 1, 4, 21) *							
0.0	1.272E − 3	1.290E − 4	1.063E − 5	0.0	0.0	0.0	0.0
0.0	7.337E − 3	0.0	4.412E − 5	0.0	0.0	0.0	0.0
4.281E − 3	7.035E − 3	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	2.004E − 4	6.626E − 4	0.0	0.0
0.0	0.0	0.0	0.0	1.437E − 3	1.696E − 3	2.511E − 4	0.0
0.0	0.0	0.0	0.0	7.528E − 3	0.0	0.0	0.0
1985 (Banks 1, 3, 21) *							
0.0	6.686E − 4	1.132E − 3	0.0	0.0	0.0	0.0	0.0
0.0	4.674E − 4	7.927E − 4	0.0	0.0	0.0	0.0	0.0
4.764E − 3	9.317E − 5	0.0	0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	3.240E − 4	1.131E − 4	3.958E − 4	0.0
0.0	0.0	0.0	0.0	2.265E − 4	7.707E − 5	2.805E − 4	0.0
0.0	0.0	0.0	0.0	7.282E − 3	0.0	0.0	0.0

* See Table 5 for the identities of these banks.
E − x means x represents the number of places to the right of the decimal point. E. g., 4.281E − 3 = 0.004281 where the last number results from rounding up the subsequent numerals in the computer printout.

Table 7
CCR cone ratio DEA results, 1984 and 1985

NO.	Bank title	Efficiency score	Reference banks				Sum of slacks (in $1 000 000)
1984							
1	BANKERS TRUST CO	1.0000	1				0
2	CITIBANK N A	1.0000	1	2			0
3	MORG. GUARANTY TR CO NY	0.9985	1	2			753.34
4	WACHOVIA BK and TR CO NA	1.0000	1	4	21		0
5	INTERFIRST BK AUSTIN NA	0.9987	12	21		0.92	
6	TX COMM BK AUSTIN NA	0.9865	12	21		9.51	
7	FIRST CITY BK OF DALLAS	0.9997	21				5.0
8	INTERFIRST BK DALLAS NA	0.9623	2				133.09
9	MBANK DALLAS NA	0.9512	1	2			51.89
10	REPUBLIC BK DALLAS NA	0.9289	2				244.49
11	INTERFIRST BK FT WOR NA	0.9650	2	12	21		16.14
12	TX AMER. BK FT WOR. NA	1.0000	1	2	12		0
13	ALLIED BK OF TX	0.8556	1	2			70.81
14	CAPITAL BK NA	0.9943	21				42.71
15	FIR CITY NAT BK HOUSTON	0.9632	1	2			91.00
16	INTERFIR BK HOUSTON NA	0.9880	2	12	21		3.41
17	REPUBLIC BK HOUSTON NA	0.7208	2	21			53.17
18	TX COMMERCE BK NA	0.9260	2				207.65
19	FROST NAT BK SAN ANTONIO	0.9000	21				27.15
20	NB OF COMM SAN ANTONIO	0.9513	2	21			4.97
21	FIRST INTRST BK NEVADA NA	1.0000	1	2	12	21	0
1985							
1	BANKERS TRUST CO	1.0000	1				0
2	CITIBANK N A	0.9575	21				1 023.70
3	MORG. GUARANTY TR CO NY	1.0000	1	3			0
4	WACHOVIA BK and TR CO NA	0.9505	21				46.80
5	INTERFIRST BK AUSTIN NA	0.7949	21				18.51
6	TX COMM BK AUSTIN NA	0.9915	21				8.61
7	FIRST CITY BK OF DALLAS	0.9947	21				4.07
8	INTERFIRST BK DALLAS NA	0.9925	1	21			77.24
9	MBANK DALLAS NA	0.8375	21				64.53
10	REPUBLIC BK DALLAS NA	0.8260	21				173.53
11	INTERFIRST BK FT WOR NA	0.9017	1	3	21		1.32
12	TX AMER. BK FT WOR. NA	0.9213	21				20.33
13	ALLIED BK OF TX	0.6632	21				79.61
14	CAPITAL BK NA	0.7864	21				72.82
15	FIR CITY NAT BK HOUSTON	0.9812	21				100.25
16	INTERFIR BK HOUSTON NA	0.9812	21				30.70
17	REPUBLIC BK HOUSTON NA	0.7254	21				41.66
18	TX COMMERCE BK NA	0.9034	21				569.91
19	FROST NAT BK SAN ANTONIO	0.7411	21				390.84
20	NB OF COMM SAN ANTONIO	0.9794	21				3.42
21	FIRST INTRST BK NEVADA NA	1.0000	1	21			0

* Cone ratio defining bank for indicated year.

Guaranty in 1984 it is nevertheless characterized as inefficient in both Tables 5 and 7 and hence is *not* used to evaluate any other bank. Citibank, on the other hand, is accorded excellent status for 1984 in Table 7 even though we omitted it in our initial set of 3 banks. Note that this bank is also used by DEA to evaluate the performances of *other* banks as can be seen by its appearance in the Reference Bank columns in 1984 in Table 7. However, this status is lost in 1985 and, unlike the situation in Table 5 Citibank is not used as a Reference Bank to evaluate the 1985 performances of other banks in Table 7.

Comparing Table 5 with Table 7 in more detail also proves illuminating. In Table 7, the banks with questionable performances which subsequently developed real trouble are all eliminated from the efficient bank groups in 1984 and 1985. Interfirst Banks of Dallas, Fort Worth and Houston, and MBank of Dallas were all in trouble in 1984 and First City Banks in Dallas and Houston also joined the problem bank list. They are not only eliminated from the efficient bank group in Table 7 but they are shown also to have nonzero slacks as further evidence of overall inefficiency witness e.g., the last column of Table 7 where the nonzero slacks for Interfirst of Dallas totaled to \$133 000 000 in 1984 and First City National Bank of Houston in 1985 totaled to \$100 250 000.

10. Individual bank DEA reports and data retransformations

A monitoring system should be capable of supplying more detail on individual banks whenever this is required. This can be supplied from our DEA analyses in a manner that we illustrate for Bank 11 (Interfirst of Fort Worth) which we use as our example for the following development. Table 7 shows that this bank was inefficient in 1995 by virtue of (i) $\theta^* = 0.9017$, which is less than unity, and (ii) the presence of nonzero slack which, as shown in the last column of Table 7, sums to $\$1.32 \times 10^6$.

To interpret these results we need to refer these solutions values from our cone ratio conversions back to the original data. We therefore introduce the following development to show how this may be accomplished in general: Let (θ^*, λ^*) be an optimal solution to (12) and let (α^*, γ^*) be an optimal solution to (10). Then

$$\omega^* = A^T\alpha^* \text{ and } \mu^* = B^T\gamma^* \tag{17}$$

will be an optimal solution to (1). See Appendix A.

Next, we represent the slack vectors associated with this optimal solution via

$$s^{+*} = (BY)\lambda^* - BY_0, \tag{18}$$

$$s^{-*} = -(AX)\lambda^* + \theta^*(AX_0). \tag{19}$$

Let

$$\tilde{s}^{+*} = Y\lambda^* - Y_0, \tag{20}$$

$$\tilde{s}^{-*} = -X\lambda^* + \theta^* X_0, \tag{21}$$

then we have $s^{-*} = A\tilde{s}$ and $s^{+*} = A\tilde{s}^{+*}$. Our development here is completed by noting that s^{+*} and

DEA MODEL:CONE-RATIO CCR MODEL WITH CONVERSION

DECISION MAKING UNIT: 11 1985 INTERFIRST OF FORT WORTH

EFFICIENCY: 0.9017

IN MILLION DOLLARS	ACTUAL	VALUE IF EFFICIENT	POTENTIAL IMPROVEMENT	SUM OF IMPROVEMENT
***********				
OUTPUTS				
***********				
INTINCOME	111.61	111.61	0.00	
NONINTINC	20.38	20.38	0.00	
ALLOWANCE	16.30	16.30	0.00	
NETLOANS	970.43	970.43	0.00	0.00
***********				
INPUTS				
***********				
INTEXPENS	91.51	79.51	12.00	
NONINTEXP	26.61	11.49	15.12	
PROVISION	7.00	11.70	-4.70	
DEPOSITS	1227.41	1227.41	0.00	22.42
				\$22.42 MILLIONS

Fig. 3. DEA printout for cone ratio CCR model – Interstate Bank of Fort Worth, 1985.

s^{-*} in Eqs. (18) and (19) are always nonnegative when the (cone ratio) transformed data are used but this is not necessarily true for $\bar{s}^{+*}$ and s^{-*} which are associated with the original data as specified in the X and Y matrices.

Fig. 3 provides an example of the kind of individual bank information which can be called up from a monitoring system like the one we are illustrating. As noted in the top line of the figure, this is the Cone Ratio CCR Model after the slacks have been converted in the just indicated manner. As shown under the Potential Improvement column of Fig. 3, an efficient performance would have required *reducing* all of the input flows except the Provision for Loan Losses. The negative value shown for the latter account means that the Provision should have been *increased* from $\$7 \times 10^6$ to $\$11.7 \times 10^6$, thereby increasing this bank's expenses by $\$4.7 \times 10^6$.

To interpret this negative slack value in the provision for loan loss in the terms we have been considering, we return to our discussions of risk coverage. The first two input inefficiencies in the column headed "Potential Improvement" in Fig. 3 are interpreted to mean that Interest Expense and Non-Interest Expense were both too high at Interstate of Fort Worth in 1985 and hence this bank was inefficient in these flow accounts. To achieve efficiency, it would have been necessary to effect a Potential Improvement by reducing Interest Expense in the amount of $\$12 \times 10^6$ and Non-Interest Expense in the amount of $\$15.12 \times 10^6$. The value of $\$4.709 \times 10^6$ in its Provision for Loan Losses is *negative*, however, which means that the Provision of $\$7 \times 10^6$ was *too small*. Its value should have been *incremented* in the amount of $\$4.7 \times 10^6$. That is, comparison with its peers showed that its expenses were too high in the first two categories while its provision for loan losses was too low. Compared to provisions made by its peer group in this "risk coverage" account, Interstate of Nevada was short in the amount of $\$4.7 \times 10^6$ this account and hence it should have *increased* this expense item.

11. Summary and conclusion

Further detail is also available to interpret those results. For instance, reference to Table 7 shows that the (1985) evaluation of Interfirst of Forth Worth is being effected by Banks 1, 3 and 21 which are, respectively, Bankers Trust, Morgan Guaranty and First Interstate of Nevada and further detail is available from the computer printouts available in most of the DEA computer codes now in use. See Ali (1993). As can be seen, a variety of new issues have been raised in this approach which include topics like the tradeoffs to be considered in "risk coverage" and "efficiency of performance". This involves tradeoffs like those we have just associated with increases in the loanloss provisions which increases the expenses of Interstate of Fort Worth in exchange for adding to its loan loss reserves.

This brings us back to the 1988 Basel agreement which, in slightly modified form, has been adopted by Federal regulatory agencies such as the Federal Reserve Board, the Office of the Comptroller of the Currency and the Federal Deposit Insurance Corporation. See Keeton (1989) for detailed discussions. In this approach to risk coverage, a set of lower bounds is first prescribed for relations which are to be maintained between tangible net worth and certain classes of assets and liabilities. Second, in determining whether these bounds are satisfied the reported values of various asset classes are required to be augmented by percentages fixed in ranges from 0 to 100% to allow for the risks that they supposedly involve. Third, the adequacy of risk coverage is then determined by calculating ratios between the former and the latter with individual banks required to satisfy a prescribed lower bound.

As can be seen, these approaches flowing from the Basel accord are very rigid. Indeed, as noted in Grenadier and Hall (1995) the risk weights that are used fail even in their limited goal of correctly quantifying "credit risk" and, in fact, no attempt is made to estimate actual risks in the sense of evaluating the chance of occurrence of undesirable events such as the bank failures or insolvency with which these regulatory agencies are concerned. The emphasis in the accords is rather on "risk coverage" by reference to results derived deterministically from reported data. Moreover, variations between periods or evidence secured from superior performances of individual banks are not used to evaluate the performances of other banks. Only prespecified asset categories are adjusted in prescribed percentages for use in effecting these evaluations of risk coverage. Finally, risk coverage is evidently the only concern in these treatments since nothing is utilized in the way of operating account information that might be used to judge efficiency or even whether efficiency has any bearing on "risk coverage".

A division of labor might suggest that "risk" evaluations at individual banks might best be undertaken in field examinations. Provisions for "risk coverage" which are "out of line" with those provided by other banks might then be used in a monitoring system to designate which banks are candidates for such examinations. This is one aspect of what our DEA approach can be designed to do. It can also be extended to include efficiency as well as risk coverage aspects of performance and the dual variables embedded in our analysis can also be used for guidance in determining the substitution (= tradeoff) possibilities that might be considered en route to regulatory actions.

This brings us to the subject of bounding techniques like those we discussed in our summary of Assurance Region approaches. As noted in Cooper et al. (1996) the AR approaches we described earlier in this paper can be used to expand the "allocative efficiency" approaches which are intended to deal with tradeoff and substitution possibilities like those that might be considered in our proposed monitoring systems. See the article by Schaffnit, Rosen and Paradi in this issue of the *European Journal of Operational Research* which shows how the AR approaches generalize the concept of "allocative efficiency" and replace the need for "exact" price-cost information in favor of lower and upper bounds on their possible values. Moreover, Arnold et al. (1997) provide yet another approach by introducing bounds on the variable values in the primal (envelopment) model of DEA. This is pertinent to the kinds of lower bounds that are prescribed in the Basel agreement, with nonsolvability occurring when these bounding values cannot be satisfied.

We now conclude by pointing to the need for further research which can join present approaches in DEA such as cone ratio envelopments and assurance region uses on dual (multiplier) problems with bounds in the primal (envelopment) models of DEA. The resulting developments could considerably enhance the powers and extend the uses of DEA in many directions besides the ones we have considered in the present paper.

Appendix A

A.1. Relations between cone forms

There are two ways of representing polyhedral cones:

(I) half space form:

$$W^1 = \{w : Cw \geq 0\}$$

$$\tilde{W}^1 = \{w : Cw \geq 0, w \geq 0\}$$

where C is $m \times n$.

(II) finite generator form:

$$W^2 = \{w : w = A\alpha, \alpha \geq 0\}$$

where A is $k \times s$.

We wish to provide relationships between A and C which can be used to analyze situations that occur in DEA analyses. All proofs of the following results are in Charnes et al. (1991).

Theorem 1.

(i) Suppose C is of full column rank, then taking $A = (C^{T}C)^{-1}C^{T}$, we have $W^1 \subseteq W^2$.

(ii) Suppose C is of full row rank, then taking $A = C^{T}(CC^{T})^{-1}$, we have $W^2 \subseteq W^1$.

For polyhedral cone ratio DEA problems we need to consider "halfspace" cones of the form

$$\tilde{W}^1 = \{w : Cw \geq 0, w \geq 0\}$$

For these we have the additional property $A \geq 0$ as given by the following two corollaries and Theorem 2 and its corollary.

Corollary 1. *Suppose C is of full row rank, and $W^1 = \tilde{W}^1$, then taking $A = C^{T}(CC^{T})^{-1}$, we have $A \geq 0$.*

Corollary 2. *Suppose C is $n \times n$ and has an inverse. Then taking $A = C$, we have*

(i) $W^1 = W^2$.

(ii) Furthermore, if $W^1 = \tilde{W}^1$, then $A \geq 0$.

Theorem 2. *Suppose C is of full column rank $C^T = (C_{11}^T, C_{21}^T)$ where C_{11} has an inverse and $C_{21}C_{11}^{-1} \geq 0$. Then taking $A = C_{11}^{-1}$, we have $W^1 = W^2$.*

Corollary. *Suppose C is of full column rank and $C_{21}C_{11}^{-1} \geq 0$. If $W^1 = \tilde{W}^1$, then taking $A = C_{11}^{-1}$, we have $A \geq 0$.*

Theorem 3.

(i) Suppose A is of full row rank, then taking $C = A^T(AA^T)^{-1}$, we have $W^1 \subseteq W^2$.

(ii) Suppose A is of full column rank, then taking $C = (A^TA)^{-1}A^T$, we have $W^2 \subseteq W^1$.

Corollary.

(i) Suppose A is of full row rank and $A \geq 0$. Then taking $C = (A^TA)^{-1} A^T$, we have $\tilde{W}^1 = W^1$.

(ii) Suppose $A \geq 0$ is $s \times s$ and has an inverse. Then taking $C = A^{-1}$, we have $\tilde{W}^1 = W^2$.

Note that the above corollary means that if $A \geq 0$, the $\tilde{W}^1$ form (special cases of which appear in the Thompson–Thrall "assurance" region form of the polyhedral coneratio CCR form) is equivalent to the corresponding W^2 finitely generated form of the CCR polyhedral coneratio form.

As mentioned, these hypotheses on the rank of C or A are often satisfied in practical applications in DEA analyses.

For example, an "assurance" region given by

$$a_{ij} \leq \frac{w_i}{w_j} \leq b_{ij}, \quad i<j, \quad i,j = 1, \ldots, n, \tag{A1}$$

which satisfies $b_{ij} \geq a_{ij}$ with at least one $b_{1k} > a_{1k}$ and $b_{1j} > 0$, can be written as

$$-w_i + b_{ij}w_j \geq 0, \tag{A2.1}$$

$$w_i - a_{ij}w_j \geq 0, \quad i<j, \quad i,j = 1, \ldots, n. \tag{A2.2}$$

For example, consider $n = 2$, i.e.,

$$a_{12} \leq \frac{w_1}{w_2} \leq b_{12}, \quad b_{12} > a_{12} \geq 0. \tag{A3}$$

Since $n(n-1) = 2$, we have

$$C = \begin{bmatrix} -1 & b_{12} \\ 1 & -a_{12} \end{bmatrix}, \tag{A4}$$

then

$$C^{-1} = \frac{1}{b_{12} - a_{12}} \begin{bmatrix} a_{12} & b_{12} \\ 1 & 1 \end{bmatrix}. \tag{A5}$$

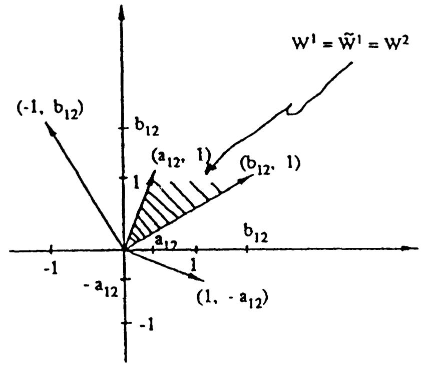

Fig. A.1.

Since $b_{12} > a_{12} \geq 0$ and $\tilde{W}^1 = W^1$, we have by the corollary of Theorem 1 or 2 that

$$A = C^{-1} > 0. \tag{A6}$$

See Fig. A.1.

A.2. Relations between optimal solution quantities

Since in many cases (e.g., Charnes et al. (1990)) we may wish to proceed to a polyhedral cone ratio form from a CCR ratio form in order to better evaluate efficiency, and since the solution of the cone ratio form is that of a CCR form with input–output data transformed by matrices which generate the cones, it is desirable to be able to recover the solution data in its original form.

Thus, consider the cone ratio DEA form (see, e.g., Charnes et al. (1989, 1990)).

$$\begin{aligned} V_p &= \text{Max } \mu^T Y_0 \\ \text{s.t.} & \\ & -\omega^T X + \mu^T Y \leq 0, \\ & \omega^T X_0 = 1, \\ & \omega \in V, \ \mu \in U. \end{aligned} \tag{A7}$$

Its dual is

$$\begin{aligned} V_D &= \text{Min } \theta \\ \text{s.t.} & \\ & -X\lambda + \theta X_0 \in -U^*, \\ & Y\lambda - Y_0 \in -U^*, \\ & \lambda \geq 0, \end{aligned} \tag{A8}$$

where V^* and U^* are respectively the negative polar of V and U.

Let $V = \{A^T\alpha:\ \alpha \geq 0\}$, $U = \{B^T\gamma:\ \gamma \geq 0\}$, then $V^* = \{v:\ Av \leq 0\}$, $U^* = \{u:\ Bu \leq 0\}$.

Using these V and U, Eqs. (A7) and (A8) can be transformed into

$$\begin{aligned} & V_p = \text{Max}\ \gamma^T(BY_0) \\ & s.t. \\ & -\alpha^T(AX) + \gamma^T(BY) \leq 0, \qquad \text{(A9)} \\ & \alpha^T(AX_0) = 1, \\ & \alpha \geq 0,\ \gamma \geq 0, \end{aligned}$$

and

$$\begin{aligned} & V_D = \text{Min}\ \theta \\ & \text{s.t.} \\ & -(AX)\lambda + \theta(AX_0) \geq 0, \qquad \text{(A10)} \\ & (BY)\lambda - (BY_0) \geq 0, \\ & \lambda \geq 0. \end{aligned}$$

Suppose (θ^*, λ^*) is an optimal solution of (A10) and (α^*, λ^*) is a corresponding solution of (A9). Then

$$\omega^* = A^T\alpha^*,\ \mu^* = B^T\gamma^* \qquad \text{(A11)}$$

is an optimal solution of (A7).

Let

$$s^{-*} = -(AX)\lambda^* + \theta^*(AX_0), \qquad \text{(A12)}$$

$$s^{+*} = (BY)\lambda^* - (BY_0). \qquad \text{(A13)}$$

(a) Suppose A is of full column rank. Since $s^{-*} = -(AX)\lambda^* + \theta^*(AX_0)$, we have

$$A^Ts^{-*} = -A^TAX\lambda^* + \theta^*A^TAX_0, \qquad \text{(A14)}$$

$$(A^TA)^{-1}A^Ts^{-*} = -X\lambda^* + \theta^*X_0. \qquad \text{(A15)}$$

In the same way, if B is of full column rank, we have

$$(B^TB)^{-1}B^TS^{+*} = Y\lambda^* - Y_0. \qquad \text{(A16)}$$

Let

$$\tilde{s}^{-*} = (A^TA)^{-1}A^Ts^{-*}, \qquad \text{(A17)}$$

$$\tilde{s}^{+*} = (B^TB)^{-1}B^TS^{+*}, \qquad \text{(A18)}$$

then

$$-X\lambda^* + {}^*X_0 - \tilde{s}^{-*} = 0, \qquad \text{(A19)}$$

$$Y\lambda^* - Y_0 - \tilde{s}^{+*} = 0. \qquad \text{(A20)}$$

(b) Suppose A and B are of full row rank. Let

$$\tilde{s}^{-*} = -X\lambda^* + \theta^*X_0, \qquad \text{(A21)}$$

$$\tilde{s}^{+*} = Y\lambda^* - Y_0, \qquad \text{(A22)}$$

then we have $s^{-*} = A\tilde{s}^{-*}$ and $s^{+*} + A\tilde{s}^{+*}$. Although s^+, $s^- \geq 0$, as given for the transformed data in (A12) and (A13), this may not be true for the $\tilde{s}^+$, $\tilde{s}^-$ as given for the original data in X and Y exhibited in (A21) and (A22).

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